Mathematical model of thermal mixing in the anaerobic fermentation of organic waste

Y. N. Sidyganov
*Doctor of technical sciences Saint-Petersburg State Agrarian University, Sidyganov_Yuriy@mail.ru*

E. M. Onuchin
*Candidate of technical sciences Mari State Technical University, OnuchinEM@marstu.net*

D. V. Kostromin
*Candidate of technical sciences Mari State Technical University, KostrominDV@marstu.net*

A. A. Medyakov
*Postgraduate student Mari State Technical University, MedyakovAA@marstu.net*

Abstract

The improvement of methods of organic waste anaerobic recycling is an important trend in biogas technologies development at the present time. In order to optimize constructive features of technical and technological systems used during anaerobic recycling for maintaining required temperature regime it is necessary to research heat exchange processes during all working regimes of the installation and in the whole volume of the bioreactor.

To increase biogas installations efficiency in the article [1] it is suggested using complex solution concerning mixing and heating with the use of catalytic heaters. In the suggested installation the mixture of gases escaping after combustion and produced biogas is used for bubble mixing of the fermentation substratum.

To solve the nonstationary problems existing calculus of approximations can be used especially finite difference method, elementary heat balance method, finite element method. To make the description of nonstationary heat exchange processes during heat mixing by heated gas in the bioreactor volume it is suggested using changed elementary balances method which means that the bioreactor volume is divided into elementary geographical shapes in the range of each shape the temperature is equal. Heat currents values, average for elementary time period, are taken as proportional to initial temperature gradient for the certain time period, and increase of heat volume content is proportional to increase of its temperature. It allows to restrict heat impact of bubbles with elementary volume where they are located in the initial moment of elementary time period.

To illustrate the processes happening during heat mixing the mathematical model was developed with the help of Microsoft Excel and Visual Basic.

Conclusions and Perspectives. Changed method of elementary balances allows to model temperature regime change in the whole bioreactor volume during the heat mixing by the heated gas with the help of the stationary heat and mass exchange equations. The developed mathematical model allows to optimize the suggested constructive solutions of the mixing and heating with the use of catalytic heaters according to the criterion of efficient maintenance of required temperature regime in the whole bioreactor volume.

**Keywords**: biogas, a bubble, the bioreactor, the thermal agitation, the catalytic heating, temperature balance

**Introduction**

The improvement of methods of organic waste anaerobic recycling is an important trend in biogas technologies development at the present time. The essential factor influencing the anaerobic recycling process efficiency is maintaining optimal temperature regime in the whole bioreactor volume for methane generation and anaerobic recycling. In order to optimize
constructive features of technical and technological systems used during anaerobic recycling for maintaining required temperature regime it is necessary to research heat exchange processes during all working regimes of the installation and in the whole volume of the bioreactor.

In work [2] on experimental bioreactor with the capacity of 2 m³ with a water jacket the researches of the heat exchange during free movement were carried out. To describe the heat exchange in the volume of fermentation substratum at approaching boundary layer it is used the criterion equation:

\[ \alpha = 0.398 \frac{c^{0.24}}{c^{0.6}} \rho^{0.24} \beta^{0.24} \frac{\Delta T^{0.24}}{g^{0.24}} \mu^{0.24} \rho^{0.28}, \]  

(1)

where \( \alpha \) - coefficient of heat diffusivity, m²/sec, \( c \) – coefficient of heat capacity, kJ/(kg*K), \( \rho \) - density, kg/m³, \( \lambda \) - coefficient of heat conductivity, W/(m*K), \( \beta \) - temperature coefficient, K⁻¹, \( \Delta T \) - temperature difference, K/m, \( \mu \) - viscosity of fermentation substratum, kg/(m*sec), \( g \) – acceleration of gravity, m/sec², \( l \) – bioreactor linear dimension, m.

In the work there is also an equation describing the heat exchange process in the volume of poultry waste fermentation near a boundary layer for geometrically similar bioreactors:

\[ Nu = 0.398(GrPr)^{0.24}, \]  

(2)

where \( Nu \) – Nusselt number, \( Gr \) – Grashof number, \( Pr \) – Prandtl number.

The authors study the heat exchange process during free bioreactor cooling but they do not research the heat exchange peculiarities during important (anaerobic fermentation) technological actions of mixing, substratum loading and uploading, discharging biogas.

In work [3] the authors study the heat exchange process in the bioreactor during mixing, there are results of experimental researches of biogas installation with the bubble mixing system. There are empirical formulas obtained with the help of regression analysis to determine kinematic viscosity (\( v \)) and surface tension (\( \sigma \)):

\[ v = 6.75 + 0.03*CB - 0.02*T, m^2/c, *10^{-6}; \]  

(3)

\[ \sigma = 119,96 - 1.69*CB - 0.02*T, H/m, *10^{-3}, \]  

(4)

where \( CB \) – solids content, \( T \) – temperature.

There is also an equation describing the heat distribution process in the substratum fermentation from heat exchanging bioreactor wall for geometrically similar bioreactors:

\[ Nu = 0.15*(Gr_{\infty}*Pr_{\infty})^{0.33}*(\frac{Pr_{\infty}}{Pr_{c}})^{0.25}, \]

where \( Nu \) – Nusselt number, \( Gr_{\infty} \) – Grashof number for liquid, \( Pr_{\infty} \) – Prandtl number for liquid, \( Pr_{c} \) – Prandtl number for heat exchanging wall.

Obtained relations allow to estimate steadiness of heat distribution in the substratum fermentation from heat exchanging wall, however, it is difficult to determine temperature distribution in the whole volume of the bioreactor.

To increase biogas installations efficiency in the article [4] it is suggested using complex solution concerning mixing and heating with the use of catalytic heaters. In the suggested installation the mixture of gases escaping after combustion and produced biogas is used for bubble mixing of the fermentation substratum. For optimization constructive solutions it is necessary to research heat exchange processes in the bioreactor volume during heat mixing by heated gas.
To solve the nonstationary problems existing calculus of approximations can be used especially finite difference method, elementary heat balance method, finite element method [5, 6]. However, the use of the methods to describe the processes of heat mixing by heated gas is complicated by the following peculiarities of the described process: complex geometrical objects form participating in the heat exchange (elementary heat balance method, finite element method) and complex description of initial conditions (finite element method) connected with an enormous quantity of moving gas bubbles – getting cold heat sources and substratum movement in the bioreactor organized by them.

To make the description of nonstationary heat exchange processes during heat mixing by heated gas in the bioreactor volume it is suggested using changed elementary balances method which means that the bioreactor volume is divided into elementary geographical shapes in the range of each shape the temperature is equal. Heat currents values, average for elementary time period, are taken as proportional to initial temperature gradient for the certain time period, and increase of heat volume content is proportional to increase of its temperature. It allows to restrict heat impact of bubbles with elementary volume where they are located in the initial moment of elementary time period.

Mathematical model description

To illustrate the processes happening during heat mixing the mathematical model was developed with the help of Microsoft Excel and Visual Basic. The window view of the mathematical model in the programme Microsoft Excel is given in the picture 1.

Figure 1 – the window of the mathematical model
1 – initial temperature values of the modeling area,
2 – input data, 3 – current temperature values of the modeling area,
4 – the values table forming in the process of modeling.

The basis of the mathematical model is equation of steady-state heat transfer. The equations of heat convection (Q) and heat flow (q) from liquid to gas through a separating wall:
\[ q = k \cdot F \cdot \Delta T, \quad Q = k \cdot F \cdot \Delta T \cdot t, \quad (5) \]

where \( k \) – coefficient of heat transmission, \( \text{W/m}^2\text{K} \), \( F \) – the area of heat exchange, \( \text{m}^2 \), \( \Delta T = T_2 - T_1 \) - temperature difference, \( \text{K} \), \( t \) – time period, \( \text{sec} \).

The equations of heat flow \( (q) \) and heat convection \( (Q) \) in the substantial medium:
\[ Q = q \cdot F \cdot t; \quad q = -\lambda \cdot \text{grad}T(x,y,z), \quad (6) \]

where \( \lambda \) - coefficient of heat conductivity, \( \text{W/(m*K)} \), \( \text{grad}T(x,y,z) \) - temperature gradient, \( \text{K/m} \).

The equations of heat flow and heat convection during flow:
\[ Q = \alpha \cdot F \cdot \Delta T \cdot t; \quad q = -\lambda \cdot \text{grad}T(x,y,z), \quad (7) \]

where \( \alpha \) – coefficient of heat emission, \( \text{W/m}^2\text{K} \).

The equation of heat balance for steady-state heat transfer:
\[ \begin{align*}
Q &= c_1 \cdot G_1 \cdot (T_{11} - T_{12}); \\
Q &= c_2 \cdot G_2 \cdot (T_{21} - T_{22}).
\end{align*} \quad (8) \]

where \( c_1, c_2 \) – specific heat capacity of heating and heated media at constant pressure, \( \text{J/(kg*K)} \), \( G_1, G_2 \) – mass flow of media, \( \text{kg} \), \( T_{11}, T_{12} \) – temperatures of heating medium at the initial and final time period, \( \text{K} \), \( T_{21}, T_{22} \) – temperatures of heated medium at the initial and final time period, \( \text{K} \).

For the modeling the processes some volume of biogas installation is taken. It is not limited by the shape peculiarities of the certain bioreactor, peculiarities of its shapes can be changed during the modeling. The modeling object presents a bioreactor component limited by two vertical planes (pic. 2). The chosen volume is divided into nine cells, the parallelepiped comes out with the cell dimensions of 3x3x1, the cells can have different volume.

![Figure 2 – Modeling volume](image)

Each cell is homogeneous by structure, properties and constant form. In the modeling process out parameter is each cell temperature. In connection with the assumptions for each cell the equations of heat and mass balance, equations heat transmission through the wall and heat emission by flow, equations of heat conductivity in the substantial medium are composed. The equations allow to determine stationary heat flow between the neighbouring cells and between the cells and the environment for initial conditions representing the matrix of 3*3 temperatures at the initial time moment. At the same time heat exchange between chosen volume, sequent and preceding along the length of the bioreactor, is not taken into consideration. Sequent and preceding volumes along the length are analogous to chosen one in dimensions and processes happening, thus, in the case of their separation into nine cells in the each cell of sequent and preceding volumes the temperature values will be the same. Thereby, between the neighbouring cells of preceding, chosen and sequent volumes the heat flow will be zero.
In the process of the modeling elementary time period is chosen \((dt)\), it is less than the modeling time. For the elementary time period with the help of certain stationary heat flows between the neighbouring cells, and also between the cells and the environment the values of transferred heat quantity are calculated. Then on the basis of composed heat balances the cells temperatures are determined at the end of elementary time period.

The obtained cells temperatures values are used to determine the heat flows and the quantity of the heat transferred at the next elementary time period. This cycle of the modeling is repeated till approaching set modeling time.

In the result of consecutive modeling with the use of stationary equations (5), (6), (7), (8) during \(N\) elementary time periods \(dt\), much less than the modeling time, the state of the modeling volume (cells temperature values) will change. But the changes of the modeling volume state will have non-stationary character.

For them the equations of the heat exchange \((Q)\) and heat flow \((q)\) from liquid to gas through the separating wall (5) will be:

\[
q = k \cdot F \cdot d(T_i - T_j), \quad d^2 Q = dq \cdot dt = k \cdot F \cdot d(T_i - T_j) \cdot dt,
\]

where \(k\) – coefficient of heat transmission, W/m²*K, \(F\) – the area of heat exchange, m², \(d(T_i - T_j)\) - change of temperature difference, K, \(dt\) – elementary time period, sec.

The equations of heat flow \((q)\) and heat convection \((Q)\) in the substantial medium (6) will be:

\[
q = -\lambda \cdot d(\nabla T(x, y)), \quad d^2 Q = dq \cdot F \cdot dt = -\lambda \cdot d(\nabla T(x, y)) \cdot F \cdot dt,
\]

where \(\lambda\) - coefficient of heat conductivity, W/(m*K), \(d(\nabla T(x, y))\) - changes of temperature gradient in the cells, K/m.

The equations of heat flow and heat convection during flow (7) will be:

\[
dq = \alpha \cdot F \cdot d(T_i - T_j), \quad d^2 Q = \alpha \cdot F \cdot d(T_i - T_j) \cdot dt,
\]

where \(\alpha\) – coefficient of heat emission, W/m²*K.

The equation of heat balance (8) will be:

\[
dQ = c_1 \cdot G_1 \cdot dT_1; dQ = c_2 \cdot G_2 \cdot dT_2
\]

where \(c_1, c_2\) – specific heat capacity of heating and heated media at constant pressure, J/(kg*K), \(G_1, G_2\) – mass flow of media, kg, \(dT_1\) – changes of heating medium temperature, K, \(dT_2\) – changes of heated medium temperature, K.

For the modeling of the heat mixing by the bubble we used the formulas analogous to the equations of the heat flow and heat convection during the flow (7) and the equation of heat and mass transfer appearing during the transfer of a liquid part from one cell to another with the bubble. At the same time liquid transfer between the cells of chosen, sequent and preceding volumes is neglected because of similar schemes of liquid movements in each volume. The liquid transfer along the bioreactor axis will be seen only during the substratum loading and uploading.

The modeling process of the heat mixing is realizing in the following way. At the initial time period the bubble is in the one of low cells of the modeling area at option and has a certain size and temperature. With the help of the stationary heat and mass exchange equations the heat flows are determined connected with the heat exchange between the bubble and cell liquid and mass exchange between cells during the bubble moving. Then for the elementary time period \((dt)\) we calculated the quantity of the transferred heat and cells temperatures values considering the heat transferred between the neighbouring cells, and also between the cells and environment. After that the heat bubble position is determined in the modeling volume at the following stage of the modeling by comparing the way passed by the
bubble and the cells size. The elementary time period should be less or equal to the time of
the bubble presence in one cell to provide the bubble impact on each cell without missing any
in the modeling process.

During the heat mixing modeling in the programme Microsoft Excel the macros was
used written in Visual Basic. Its text is the following:

Sub Macros()
    Sheets("List1").Select
    Cells(3, 10).Value = Cells(2, 1).Value 'assign to current values initial conditions'
    Cells(5, 10).Value = Cells(3, 1).Value
    Cells(7, 10).Value = Cells(4, 1).Value
    Cells(3, 12).Value = Cells(2, 2).Value
    Cells(5, 12).Value = Cells(3, 2).Value
    Cells(7, 12).Value = Cells(4, 2).Value
    Cells(3, 14).Value = Cells(2, 3).Value
    Cells(5, 14).Value = Cells(3, 3).Value
    Cells(7, 14).Value = Cells(4, 3).Value
    Cells(17, 7).Value = 1 'assign to current bubble position according to the height value 1'
    Cells(17, 10).Value = 0 'assign to the way passed by the bubble value 0'
    Cells(19, 7).Value = Cells(16, 5).Value 'assign to the current bubble temperature the
values of initial conditions'
    For I = 1 To Cells(2, 5).Value 'assign to the current values of the dynamic
parameters the values of the following stage'
        A = Cells(22, 10).Value 'assign the values of the following stage for the liquid
temperature variables'
        B = Cells(23, 10).Value
        C = Cells(24, 10).Value
        D = Cells(22, 11).Value
        E = Cells(23, 11).Value
        F = Cells(24, 11).Value
        G = Cells(22, 12).Value
        H = Cells(23, 12).Value
        J = Cells(24, 12).Value
        K = Cells(17, 11).Value 'assign the value of the following bubble position according
to the height of variable'
        L = Cells(17, 14).Value 'assign the value of the bubble way at the following stage
variable'
        M = Cells(19, 11).Value 'assign the bubble temperature values at the following stage
variable'

    K = Cells(17, 11).Value 'assign the value of the following bubble position according
to the height of variable'
    L = Cells(17, 14).Value 'assign the value of the bubble way at the following stage
variable'
    M = Cells(19, 11).Value 'assign the bubble temperature values at the following stage
variable'
Conclusion

1. Changed method of elementary balances allows to model temperature regime change in the whole bioreactor volume during the heat mixing by the heated gas with the help of the stationary heat and mass exchange equations.

2. The developed mathematical model allows to optimize the suggested constructive solutions of the mixing and heating with the use of catalytic heaters according to the criterion of efficient maintenance of required temperature regime in the whole bioreactor volume.

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